2022 Moscow Workshop on Electronic and Networking Technologies (MWENT)

Analysis of Operation of the non-Energy Parameter Meter of the Useful Signal under the Influence of Additive and Multiplicative Noises

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***Abstract*—The analysis of the non-energy signal parameter tracking meter under the influence of additive white noise and multiplicative noise is carried out. The functional scheme of the two-channel discriminator with the detuned channels according to some parameter is given. The operation of the named discriminator is described. It is noted that the measured parameter is a random function of time. The occurrence of dynamic error is shown, as well as the appearance of fluctuation errors under the influence of noises. The assumption system accepted in the paper allows considering the servo-measuring device as a linear system according to the measured parameter. The analysis is done for a quadratic envelope detector. It is noted that the effective bandwidth of the tracking meter depends on the level of noises. Expressions for definition of a dispersion of errors of measurement both in the presence only additive hindrances, and at action on a signal multiplicative hindrance are received. The coefficients making it possible to estimate the influence of multiplicative noises on deterioration of accuracy of the tracking meter characteristics are introduced. It is shown that these coefficients can be calculated by using the slope of the discrimination characteristic, the spectral density of fluctuations at the discriminator output under the zero error and the coefficients defining the normalizing effect of automatic gain control in the presence and absence of noises.**

Keywords*—tracking parameter, multiplicative (modulating) noise, additive white noise, automatic gain control, spectral density of fluctuations, steepness of the discriminatory characteristic, variance of the errors of parameter measurement

I. Introduction

Automatic tracking meters for non-energy parameters are widely used in various technical devices, in particular in aviation, rocket and space technology. The construction and operation of such devices were considered in a number of works [1-3 etc.].

A generalized diagram of such a measuring device is shown in Figure 1.

A mixture $u_{in}(t)$ of the signal $u_M(l_0, t)$ distorted by multiplicative (modulating) noise (MN) and additive white noise (AN) $n_0(t)$ [4-6 etc.] enters the discriminator (D), in which a mismatch signal $\chi(\varepsilon, t)$ is discriminated, proportional to the difference between the measured l^* and true l_0 values of the parameter $\varepsilon = l^* - l_0$.

The feedback circuit is closed through a linear smoothing filter (LSF) and actuators. Controlled heterodynes, antenna servo devices, etc. can perform this function.

In real receiving devices, the amplifiers representing the meter according to the block diagram of Fig. 1 usually contain an automatic gain control (AGC) circuit, which maintains constant average level of power or the signal envelope $u_{in}(t)$ at the input of D [7].

Assuming that the changes in the average power of the input signal are slow, we will take into account the influence of the AGC by means of appropriate normalization, which ensures the constancy of the signal power at the input of the discriminator in the band of its linear circuits.

In general, the measured parameter is a random function of time. At the same time, even in the absence of AN and MN, it is measured with some error, which is called a dynamic error. In many practical problems, the correlation interval of fluctuations of the measured parameter itself is much larger than the correlation interval of the noise modulation function and additive noise. In these cases, the measured parameter can be considered constant during the whole measurement interval and only the fluctuation measurement errors caused by AN and MN can be considered.

Thus, in the case under consideration, the value $\varepsilon = l^* - l_0$ determines only the fluctuation error.

The purpose of the work is to analyze the operation of the two-channel tracking meter of the information parameter of the useful signal with disrupted channels under the influence of additive and multiplicative noises. The influence of the discriminator parameters on the degradation of the accuracy of measurement of the useful signal parameter is determined.

II. A Two-Channel Discriminator with Channels Mismatched by the Parameter *L*

We will narrow down further analysis to a discriminator diagram widely used in various systems – a two-channel discriminator with channels mismatched by the parameter l (Figure 2).

The principle of its operation is as follows. The input signal, distorted by noise $u_{in}(t)$ is sent to two multipliers, where it is multiplied by reference signals shifted by $\pm\Delta_l$ of the parameter l with respect to the parameter value l^* being measured at the moment. Reference signals are generated by generators that are controlled by signals proportional to the measured value of the parameter. After multiplication, the signals are sent to filters with a frequency response $\dot{g}_F(\omega)$ (F) and then to quadratic envelope detectors (QED).

The output signal of the discriminator (error signal) is proportional to the difference between the squares of signal envelopes at the output of filters.

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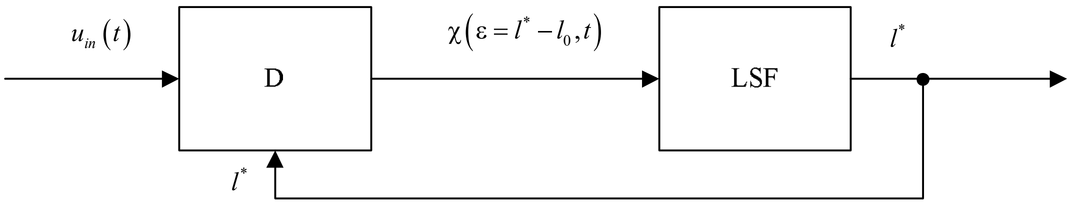

Fig. 1. Generalized block diagram of a tracking measuring device

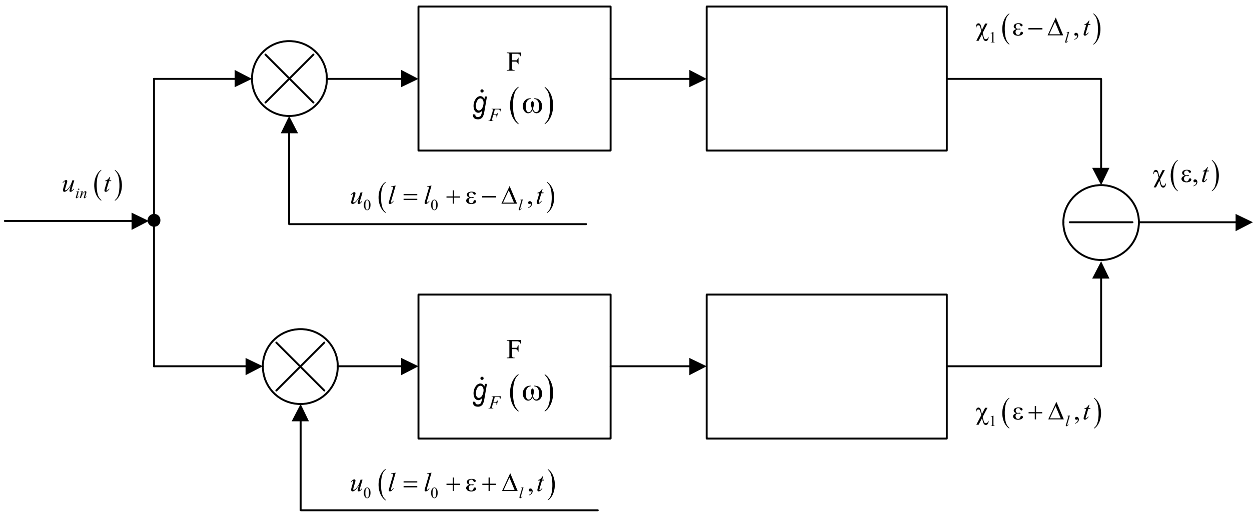

Fig. 2. Functional diagram of a two-channel discriminator with channels mismatched by the parameter l

The coefficients k_{AGC} and k_{0AGC}, that determine the normalizing effect of the AGC and keep the input signal power constant in the band of linear circuits of the discriminator in the considered discriminator circuit can be defined as values proportional to the sum of squares of the envelopes at the output of the filters.

Further on, the proportionality coefficient will be considered equal to unity. At the same time, in the absence of noise, the signal at the input of linear circuits of the discriminator, taking into account the normalizing influence of the AGC, will have a single-unit power.

III. Evaluation of Effect of Multiplicative Noise on Accuracy Characteristics of Tracking Meter

In most cases, when it is necessary to take into account the influence of the AN and MN on the measurement result, the width of the spectrum of fluctuations of the output signal (voltage) D $\chi(\varepsilon, t)$ is much larger than the effective bandwidth of the closed loop of the meter. This makes it possible to consider the spectral power density of fluctuations of the output signal D constant within the bandwidth of the tracking system and equal to the spectral power density of fluctuations at zero frequency. As for the case of a relatively low level of noise, when the characteristic D for the measured parameter – the measurement error ε – can be considered linear, the output voltage D is approximately written as

$$\chi(\varepsilon, \tau) = \chi_1(\varepsilon - \Delta_l, \tau) - \chi_2(\varepsilon + \Delta_l, \tau), \tag{1}$$

where $\chi_{1.2}(\varepsilon \pm \Delta_l, \tau)$ is the steepness of the discriminatory characteristic, G_χ is a spectral density of fluctuations of the output signal D $\chi(\varepsilon, t)$ at zero frequency when $\varepsilon = 0$, $n_0(t)$ is white noise with a single spectral density.

In accordance with (1), the spectral density of fluctuations of the output signal D does not depend on the current value of the measurement error ε, it coincides with the first term in the measurement error ε expansion of the real density distribution into a Taylor series.

The analysis carried out in [3] shows that such a representation of the spectral density is valid if the following conditions are met:

- measurement errors are small and do not go beyond the linear section of the characteristic D;
- fluctuations of the received signal, caused by MN for example, have a correlation interval significantly smaller than the time constant of the tracking meter, the width of the spectrum of signal fluctuations is significantly larger than the width of the spectrum of the measured parameter.

As a rule, the latter condition is almost always fulfilled for MN, which leads to noticeable distortions of the signal during the time equal to the time of its coherent processing in the receiver's linear circuits, since this time is significantly less than the time constant of the tracking meter.

Thus, the adopted system of assumptions regarding the characteristics of the received signal and noise leads to the analysis of the tracking meter as a linear system in terms of the measured parameter. Such consideration is permissible only at a low level of AN and MN, when measurement errors do not go beyond the linear section of the characteristic D.

It was pointed out above that in the case when the average input power of the signal changes slowly over a time equal to the time constant of linear circuits D, the influence of AGC circuits can be taken into account by some normalizing coefficient k_{AGC} proportional to the power of the input signal $u_{in}(t)$, for example.

Taking into account the normalizing influence of the AGC, the input signal D can be written as $\frac{u_{in}(t)}{k_{AGC}^{0.5}\left[u_{in}(t)\right]}$.

Since the characteristics D introduced above for the measured parameter k_d, G_χ depends on the absolute level of the input signal D, it is necessary to take into account the influence of the modulating action of the AGC on these characteristics.

Further, we will consider D with a quadratic envelope detector. The output signal of such D is proportional to the power of the input signal. In this case, taking into account the influence of the AGC, the steepness of the discriminatory characteristic and the spectral density of fluctuations at the output D are written as

$$k_{d.AGC} = \frac{k_d}{k_{AGC}\left[u_{in}(t)\right]}, \quad G_{\chi.AGC} = \frac{G_\chi}{k_{AGC}^2\left[u_{in}(t)\right]}. \tag{2}$$

The variance of the errors of parameter measurement by the tracking meter is equal to the total power of the fluctuations of the output signal D in the bandwidth of the closed tracking system divided by the square of the slope of the discrimination characteristic k_d^2.

If $\Delta\Omega_{ts}$ is the efficient width of the frequency characteristic of the closed tracking system of the meter $G_c(j\omega)$ in terms of the square of the module, then the variance of measurement errors of the parameter l at a low level of noise, taking into account (1), is determined by the expression

$$\sigma_l^2 = \frac{G_\chi}{k_d^2}\Delta\Omega_{ts}, \tag{3}$$

where

$$\Delta\Omega_{ts} = \int_{-\infty}^{\infty}\frac{\left|G_c(j\omega)\right|^2}{\left|G_c(0)\right|^2}d\Omega. \tag{4}$$

The frequency-response characteristics of a closed-loop tracking system $G_c(j\omega)$ is determined by the frequency-response characteristics of an open-loop tracking system $G_o(j\omega)$ using the transformation

$$G_c(j\omega) = \frac{k_0 k_{d.AGC} G_o(j\omega)}{1 + k_0 k_{d.AGS} G_o(j\omega)}, \tag{3a}$$

where k_0 is a coefficient proportional to the gain of the open tracking system.

Suppose that the smoothing circuits of the tracking meter are constructed in such a way that, for given dynamic errors and the nominal value of steepness $k_{dn.AGC}$ the minimum value of the bandwidth of the tracking meter is provided, which is equal to $\Delta\Omega_{ts.0}$.

Note that the nominal steepness of D $k_{dn.AGC}$ is defined as the steepness of D in the absence of AN and MN. Similarly, we will determine the nominal values of other parameters, denoting them with the index "n", for example $k_{n.AGC}$.

Since the steepness of the discrimination characteristic depends on the level and parameters of AN and MN, in accordance with (3a), (4), the effective bandwidth of the meter as a closed tracking system also depends on the level of noise.

IV. DEFINE VARIANCE OF MEASUREMENT ERRORS

If the above condition is fulfilled regarding the choice of the bandwidth of the smoothing circuits of the meter in the absence of noise the following expressions can be obtained for the effective width of the frequency-response characteristics $G_c(j\omega)$ in the presence of noise for various smoothing filters:

- for a smoothing filter in the form of a single integrator or RC circuit

$$\Delta\Omega_{ts} = \Delta\Omega_{ts.0}\frac{k_{d.AGC}}{k_{dn.AGC}}; \tag{5}$$

- for a smoothing filter in the form of a double integrator with correction

$$\Delta\Omega_{ts} = \frac{\Delta\Omega_{ts.0}}{2}\left(1 + \frac{k_{d.AGC}}{k_{dn.AGC}}\right); \tag{6}$$

- for a smoothing filter in the form of two RC circuits with correction

$$\Delta\Omega_{ts} = \frac{\Delta\Omega_{ts.0}}{3}\frac{1 + \frac{k_{d.AGC}}{2k_{dn.AGC}}}{1 + \frac{k_{dn.AGC}}{k_{d.AGC}}\sqrt{\frac{2\left(T_1+T_2\right)^2}{k_0 k_{d.AGC} T_1 T_2}}}, \tag{7}$$

where T_1, T_2 are time constants of RC circuits.

Note that in the practical diagrams of tracking meters, the value

$$y = \sqrt{\frac{2\left(T_1+T_2\right)^2}{k_0 k_{d.AGC} T_1 T_2}},$$

included in (7) is much less than unity [9].

If the noise level is low, the product $y\frac{k_{dn.AGC}}{k_{d.AGC}}$ will also be much less than unity. In this case, the expression defining $\Delta\Omega_{ts}$ for the smoothing filter in the form of two RC circuits with correction does not explicitly depend on the time constants of the RC circuits and is equal to

$$\Delta\Omega_{ts} = \frac{\Delta\Omega_{ts.0}}{3}\frac{1 + \frac{k_{d.AGC}}{2k_{dn.AGC}}}{1 + \frac{k_{dn.AGC}}{k_{d.AGC}}\sqrt{\frac{2\left(T_1+T_2\right)^2}{k_0 k_{d.AGC} T_1 T_2}}}. \tag{7a}$$

If we denote the variance of measurement errors in the presence of only AN (MN is absent) as $\sigma_{l.0}^2$, taking into account (5)-(7a), the variance of measurement errors of the parameter l (3) in the presence of AN and MN can be written as:

- with a smoothing filter in the form of an integrator or an RC circuit

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{k_{d.AGC}}{k_{d.0AGC}}; \qquad (8)$$

- with a smoothing filter in the form of a double integrator with correction

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{1+\dfrac{k_{d.AGC}}{k_{dn.AGC}}}{1+\dfrac{k_{d.0AGC}}{k_{dn.AGC}}}; \qquad (9)$$

- with a smoothing filter in the form of two RC circuits with correction

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{1+\dfrac{k_{d.AGC}}{2k_{dn.AGC}}}{1+\dfrac{k_{d.0AGC}}{2k_{dn.AGC}}}, \qquad (10)$$

where $G_{\chi 0}$, $k_{d.0}$ is a spectral density of the discriminator output signal and the steepness of the discrimination characteristic in the presence of only AN and without taking into account the normalizing action of the AGC.

Note that in accordance with the above definitions

$$k_{d.AGC} = k_d / k_{AGC}; \quad k_{\text{д}.0.\text{АРУ}} = k_{\text{д}.0} / k_{0.\text{АРУ}};$$
$$k_{dn.AGC} = k_{dn} / k_{n.AGC},$$

where $k_{0\text{AGC}}$ is a coefficient characterizing the normalizing effect of AGC in the presence of only AN.

It follows that $k_{dn} = k_{d.0}$. Then instead of (8)-(10), respectively, we get

- for one RC circuit, one integrator

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{k_{0.AGC}}{k_{AGC}} = \sigma_{l.0}^2 \eta_{l_{\text{M}.1}}; \qquad (8a)$$

- for two integrators with correction

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{1+\left(k_{n.AGC}/k_{AGC}\right)\left(k_d/k_{d.0}\right)}{1+k_{n.AGC}/k_{0.AGC}} = \qquad (9a)$$
$$= \sigma_{l.0}^2 \eta_{l_{\text{M}.2}};$$

- for two RC circuits with correction

$$\sigma_l^2 = \sigma_{l.0}^2 \frac{G_\chi}{G_{\chi.0}} \frac{k_{d.0}^2}{k_d^2} \frac{1+\left(k_{n.AGC}/k_{AGC}\right)\left(k_d/2k_{d.0}\right)}{1+k_{n.AGC}/2k_{0.AGC}} = \qquad (10a)$$
$$= \sigma_{l.0}^2 \eta_{l_{\text{M}.3}}.$$

Thus, the deterioration of the accuracy characteristics of the tracking meter due to the influence of MN in comparison with the case when they are absent can be estimated by the coefficients $\eta_{l_{\text{M}.i}}$ when $\sigma_{l.0}^2$ in (8a)-(10a).

In order to calculate these coefficients, it is sufficient to determine the characteristics of the discriminator: the steepness of the discriminatory characteristic (k_d, k_{d0}) and the spectral density of fluctuations at the output of the discriminator at zero error (G_χ, $G_{\chi 0}$), as well as the coefficients determining the normalizing effect of the AGC (k_{AGC}, $k_{0.\text{AGC}}$, $k_{n.\text{AGC}}$) in the presence and in the absence of noise.

V. Conclusion

The operation of a two-channel discriminator with measurement error detuned channels is described. The named discriminator is, in turn, a key component of a tracking meter for non-energy parameters of a useful signal. The influence of both additive white noise and multiplicative noise on the discriminator is considered.

Expressions for the steepness of the discriminatory characteristic and the spectral density of fluctuations at the discriminator output under the action of automatic gain control are obtained. The variance of the parameter measurement at a low level of noise is determined. It is shown that the effective bandwidth of the tracking meter depends on the level of noise, for which also. Expressions for determining the said bandwidth for various smoothing filters are found. The expressions for determination of the measurement error dispersion both in the presence of additive noises only and in the presence of multiplicative noises, which take into account the normalizing effect of AGC are obtained. The coefficients are introduced to estimate the influence of noises on the degradation of the accuracy of the tracking meter characteristics. To calculate these coefficients the steepness of the discriminative characteristic, the spectral density of fluctuations on the discriminator output at zero error and "AGC normalizing coefficients" both in the presence and in the absence of noise should be used.

References

[1] G. L. Charvat Small and Short-Range Radar Systems. CRC Press, 2014.

[2] Ivan Saetchnikov, Victor Skakun, and Elina Tcherniavskaia, “Efficient objects tracking from an unmanned aerial vehile”, Proceedings of 2021 IEEE 8th International Workshop on Metrology for AeroSpace (MetroAeroSpace). Naples, Italy, 23-25 June 2021. DOI: 10.1109/MetroAeroSpace51421.2021.9511748

[3] Jian Lan, and X. Rong Li, “Tracking of Extended Objects with High-Resolution Doppler Radar,” IEEE Trans. on Aerospace and Electronic Systems, 2016, Volume 52, Issue 6, pp. 2973–2989. https://doi.org/10.1109/TAES.2016.130346

[4] V. M. Artyushenko, and V. I. Volovach, “Synthesis of Algorithms of Adaptive Signal Processing Using of the Nonlinear Blocks with Approximation of Optimal Amplitude Transfer Characteristic”, 2019 Siberian Conference on Control and Communications (SIBCON). Proceedings. Tomsk State University of Control Systems and Radioelectronics, Russia, Tomsk, April 18-20, 2019. DOI: 10.1109/SIBCON.2019.8729668

[5] S. Kassam, Signal Detection in Non-Gaussian Noise. Berlin: Springer, 1988.

[6] E. Palahina, and V. Palahin, “Signal detection in additive-multiplicative non-Gaussian noise using higher order statistics”, 2016 26th International Conference Radioelektronika (RADIOELEKTRONIKA), IEEE, 2016, pp. 262-267. DOI: 10.1109/RADIOELEK.2016.7477367

[7] V. M. Artyushenko, and V. I. Volovach, “Analysis of Statistical Characteristics of Probability Density Distribution of the Signal Mixture and Additive-multiplicative non-Gaussian Noise”, 2019 Dynamics of Systems, Mechanisms and Machines (Dynamics), 2019, pp. 1-6. DOI: 10.1109/Dynamics47113.2019.8944670

[8] V. M. Artyushenko, and V. I. Volovach, “Comparative analysis of discriminators efficiency of tracking meters under influence of non-Gaussian broadband and band-limited noise”, XI International IEEE Scientific and Technical Conference “Dynamics of Systems, Mechanisms and Machines (Dynamics)”, Proceedings. Omsk: Omsk State Technical University, Omsk, Russia, Nov 14-16, 2017. DOI: 10.1109/Dynamics.2017.8239430

[9] V. Krishnan, Probability and Random Processes; 2nd ed.; Wiley, 2016,