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MEASURING AND INFORMATION-MEASURING SYSTEMS

Estimation of Motion Intensity of Extended Objects Using Generalized Weibull Distribution

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Abstract—The estimation of the scale parameter of the generalized Weibull probability density function by the criterion of the conditional risk minimum is considered and analyzed. Optimal estimates of the envelope intensity of the signals reflected from extended objects are obtained, provided that the intervals in the motion of these objects are independent random variables. These estimates are compared with the best unbiased estimate for various loss functions. It is shown that the convergence of the risk of the best unbiased estimate to the risk of the optimal estimate for the loss function, equal to the error module, is faster than the quadratic loss function. It is noted that the optimal estimates for the invariant and power loss functions coincide. The lower and upper ε-confidence boundaries are obtained and the conditions under which the obtained estimates correspond to the estimation of the Weibull distribution parameter are determined.

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1. INTRODUCTION

It is of significant interest in many applied problems to determine the value of intensity of random streams in which the intervals between events are independent random values. In particular, such problem is topical for systems of automatic speed control of railway cuts [1, 2] related with the determination of intensity of motion of the latter at their runaway at hump yards. Note that the intensity of motion at the hump yard is usually determined by the intervals between the signals, reflected from the cuts when they fill classification tracks, arriving from the radar speed meter [3].

The performed studies show that the signal reflected from the cuts (extended object) is well described by the multibeam model [4, 5] and has the form of an amplitude-modulated oscillation. Here, the depth of signal modulation varies within large boundaries and may achieve 100% [6, 7].

To determine the probabilistic characteristics of intensity of intervals between the cuts rolling at the hump yard, it is more convenient to use not the reflected signal itself, but its envelope $U(t)$. In this case, when we statistically process the experimental data of the envelope of the signal reflected from an extended object, we may use the so called generalized Weibull distribution law for describing probabilistic models:

$$W_\theta(U_i) = \begin{cases} \Gamma(\gamma_i)^{-1}(\theta\mu_i)^{\lambda\gamma_i}\lambda U_i^{\lambda\gamma_i}\mathrm{e}^{-(\theta\mu_i U_i)^\lambda}, & U_i \geq 0; \\ 0, \quad U_i < 0, \end{cases} \tag{1}$$

where $\mu_i = \Gamma(\gamma_i + \lambda^{-1})/\Gamma(\gamma_i)$, $\Gamma(\cdot)$ is the gamma function, $\lambda > 0$ and $\gamma_i > 0$ are the known constants, the distribution parameters, $\theta > 0$ is the scale parameter, and U_i is the useful signal reflected from the ith extended object.

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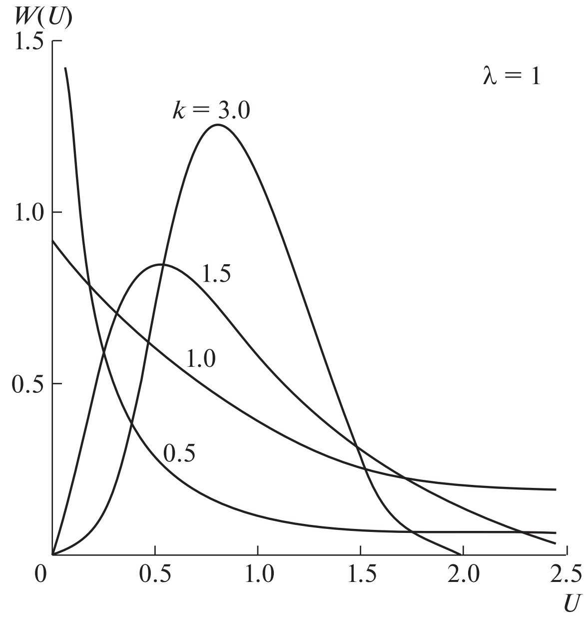

Fig. 1. Weibull probability density function for different values of k.

Note that at $\gamma_i = 1$ distribution (1) becomes the Weibull probability density functions (PDFs) with the scale parameter λ (Fig. 1) [8]

$$W(U) = \begin{cases} (k/\lambda)(U/\lambda)^{k-1}\mathbf{e}^{(-U/\lambda)^k}, & U \geq 0; \\ 0, \quad U < 0, \end{cases}$$

where $k > 0$ is the distribution parameter characterizing the form of PDF.

At $\lambda = 1$ distribution (1) transforms to the gamma distribution with γ_i degrees of freedom.

The current work is aimed at determining the value of motion intensity of extended objects using the generalized Weibull distribution under the conditions when the intervals in the motion of these objects are independent random values.

Let us consider and analyze the optimal estimation of the scale parameter θ of the generalized Weibull PDF (1).

2. POINTWISE ESTIMATION

Suppose that there are n observable independent nonnegative random values U_i, $i = \overline{1, n}$ which have the conditional PDF at fixed parameter θ; the PDF is in its turn characterized by expression (1). We need to estimate the unknown parameter $\theta > 0$ relative to the Lebesgue measure on a straight line [8].

We take that $U_{s.1}, \ldots, U_{s.n}$ are the intervals between the events of some random stream U_s, where $U_{s.i}$ is the envelope of the signal reflected from the ith extended object. Then, the problem consists in estimating the intensity of this stream by observing the inter-event times, because $M_\theta U_{s.i} = \theta^{-1}$, where M_θ is the symbol of mathematical expectation corresponding to the PDF $W_\theta(U^n) = \sum_{i=1}^{n} W_\theta(U_i)$, where $W_\theta(U_i)$ is determined by expression (1).

We introduce the estimate of the parameter θ: $\hat{\theta}_n = \hat{\theta}_n(U^n)$ obtained after observation of the sample $U^n = (U_1, \ldots, U_n)$, where $U_1, \ldots, U_n$ are the observed values $U_{s.i}$.

We think that the loss function has the form

$$f_n(\hat{\theta}_n, \theta) = g_n(\hat{\theta}_n/\theta - 1), \tag{2}$$

that is, it is invariant to scale transformations.

We introduce the conditional risk of estimating the parameter θ:

$$r_\theta(\hat{\theta}_n) = M_\theta f_n(\hat{\theta}_n, \theta).$$

We need to find such estimate $\hat{\theta}_n^0$ that uniformly minimizes $r_\theta(\hat{\theta}_n)$ over the parameter $\theta > 0$. However, we account for the fact that there exists no such estimate in the class of all possible estimates and have to narrow the class of the considered estimates.

As intermediate estimates we consider the class of estimates Ψ satisfying the property

$$\hat{\theta}_n(U^n c) = \hat{\theta}_n(U^n)/c \tag{3}$$

for all $c > 0$; $cU^n = (cU_1, \ldots, cU_n)$.

There may exist the optimal estimate in the class of estimates Ψ.

With acccount for expressions (1)–(3) we write

$$r_\theta(\hat{\theta}_n) = M_\theta g_n[\hat{\theta}_n(\theta U^n) - 1] = M_1 g_n[\hat{\theta}_n(U^n) - 1] = r_1(\hat{\theta}_n), \quad \hat{\theta}_n \in \Psi.$$

Consequently,

$$r_\theta(\hat{\theta}_n^0) = \inf_{\hat{\theta}_n \in \Psi} r_1(\hat{\theta}_n), \quad \theta > 0, \tag{4}$$

where inf is the sign denoting the interior point of the set.

It follows from expression (4) that, for existence of the optimal estimate in the class Ψ, it is sufficient that Ψ contains at least one such estimate $\hat{\theta}_n$ for which the inequality is fulfilled

$$r_1(\hat{\theta}_n) < \infty.$$

It is known [9] that the estimates having a constant conditional risk are called equivariant; therefore, the best estimate at invariant loss function (2) is the best equivariant estimate.

Note that the optimal estimate may exist not only at invariant loss functions (2), but also at the loss functions that have the following form:

$$f_n(\hat{\theta}_n, \theta) = A_n|\hat{\theta}_n - \theta|^\alpha/\theta^{\alpha-\beta} + c_n, \quad \alpha > 0, \quad \beta \geq 0, \quad 0 < A_n < \infty, \quad 0 \leq c_n < \infty. \tag{5}$$

We make use of expressions (4) and (5) and may check that

$$r_\theta(\hat{\theta}_n) = A_n\theta^\beta M_1|\hat{\theta}_n(U^n) - 1|^\alpha + c_n = \theta^\beta r_1(\hat{\theta}_n) + c_n(1 - \theta^\beta), \quad \hat{\theta}_n \in \Psi.$$

Hence,

$$\hat{\theta}_n^0(U^n) = \arg \inf_{\hat{\theta}_n \in \Psi} r_1(\hat{\theta}_n). \tag{6}$$

Thus, the estimate at loss function (5) optimal in the class Ψ coincides with the optimal equivariant estimate at the loss function

$$f_n(\hat{\theta}_n, \theta) = A_n|\hat{\theta}_n - \theta|^\alpha/\theta^\alpha + c_n,$$

corresponding, in its turn, to (5) at $\beta = 0$.

In the following, without loss in generality, we will assume that $A_n = 1$ and $c_n = 0$.

We introduce the improper prior distribution θ with the PDF $W_0^{(\beta)}(\theta) = \theta^{-(\beta+1)}$. For an arbitrary invariant loss function (2), for which (5) at $\beta = 0$ is a special case, we need to use $W_0^{(\beta)}(\theta) = \theta^{-1}$ [10].

With account for (1) and independence of observations n, the prior PDF θ is in this case determined by the equality

$$W_0^{(\beta)}(\theta) = \frac{\lambda y_n^{\vartheta_n - \beta/\lambda} \theta^{\lambda\vartheta_n - \beta - 1} \mathbf{e}^{-\theta^\lambda y_n}}{\Gamma(\vartheta_n - \beta/\lambda)}, \quad \theta > 0, \tag{7}$$

where $\vartheta_n = \sum_{i=1}^n \gamma_i > \alpha/\lambda$; $y_n = \sum_{i=1}^n (\mu_i U_i)^\lambda$.

Then, as follows from work [10], the conditional risk may be found from the expression

$$r_\theta(\hat{\theta}_n) = \theta^\beta \mathbf{M}_1 R_n^{(\beta)}(\hat{\theta}_n, Y_n), \quad \hat{\theta}_n \in \Psi, \tag{8}$$

where $Y_n = \sum_{i=1}^{n}(\mu_i U_{s.i})^\lambda$,

$$R_n^{(\beta)}(\hat{\theta}_n, Y_n) = \int\limits_0^\infty |\hat{\theta}_n - \theta|^\alpha \theta^{\beta-\alpha} W_0^{(\beta)}(\theta) d\theta \tag{9}$$

is the function of a posteriori risk and at $\beta = 0$ it is independent from the observation [10].

Therefore, optimal estimate (6) is determined following from the condition

$$R_n^{(\beta)}(\hat{\theta}_n^0, y_n) = \inf_{z>0} R_n^{(\beta)}(z, y_n). \tag{10}$$

Let us consider and analyze different loss functons.

3. QUADRATIC LOSS FUNCTION

If the loss function has the form

$$f_n(\hat{\theta}_n, \theta) = (\hat{\theta}_n - \theta)^2, \tag{11}$$

then it follows from (9)–(11) that the optimal estimate (in the sense of (6)) at $\beta = 2$ coincides with a posteriori average PDF (7).

Using expression (7) and replacing the variables $\theta^\lambda = q$, it is not hard to see that the mth moment of PDF (7) is as follows

$$\nu_m^{(\beta)}(y_n) = \frac{\Gamma(\vartheta_n - (\beta - m)/\lambda) y_n^{-m/\lambda}}{\Gamma(\vartheta_n - \beta/\lambda)}. \tag{12}$$

Setting $\beta = 2$ and $m = 1$ in (12), we obtain

$$\hat{\theta}_n^0(y_n) = \frac{\Gamma(\vartheta_n - 1/\lambda) y_n^{-1/\lambda}}{\Gamma(\vartheta_n - 2/\lambda)}. \tag{13}$$

We use expressions (12) and (13) and may show that a posteriori risk of the estimate $\hat{\theta}_n^0$ is equal to

$$R_n^{(2)}(\hat{\theta}_n^0, y_n) = \frac{[\Gamma(\vartheta_n)\Gamma(\vartheta_n - 2/\lambda) - \Gamma^2(\vartheta_n - 1/\lambda)] y_n^{-2/\lambda}}{\Gamma^2(\vartheta_n - 2/\lambda)}. \tag{14}$$

It is clear that the statistic Y_n has the gamma distribution with the parameters $(\theta^\lambda, \vartheta_n)$:

$$W_\theta(y_n) = \Gamma(\vartheta_n)^{-1} \theta^{\lambda \vartheta_n} y_n^{\vartheta_n - 1} \mathbf{e}^{-\theta^\lambda y_n}, \quad y_n \quad \geq 0. \tag{15}$$

Using expressions (8) and (13)–(15), we derive

$$M_\theta \hat{\theta}_n^0(Y_n) = \frac{\Gamma^2(\vartheta_n - 1/\lambda)\theta}{\Gamma(\vartheta_n)\Gamma(\vartheta_n - 2/\lambda)}, \tag{16}$$

$$r_\theta(\hat{\theta}_n^0) = \theta^2 \left[1 - \frac{\Gamma^2(\vartheta_n - 1/\lambda)}{\Gamma(\vartheta_n)\Gamma(\vartheta_n - 2/\lambda)}\right]. \tag{17}$$

Besides, $\vartheta_n > 2/\lambda$ is the condition of existence of the optimal estimate.

It follows from expressions (13) and (16) that the unbiased estimate has the form

$$\theta_n^*(y_n) = \frac{\Gamma(\vartheta_n/\lambda) y_n^{-1/\lambda}}{\Gamma(\vartheta_n - 1/\lambda)}. \tag{18}$$

If Y_n is the sufficient statistic for the family of PDFs $\{W_\theta(U^n)\}$ generated by (1) and family (15) is complete according to [9], then estimate (18) in the class of unbiased estimates appears to be optimal [9, 11]. Its risk may be determined by

$$r_\theta(\theta_n^*) = \theta^2 \left[\frac{\Gamma(\vartheta_n)\Gamma(\vartheta_n - 2/\lambda)}{\Gamma^2(\vartheta_n - 1/\lambda)} - 1\right]. \tag{19}$$

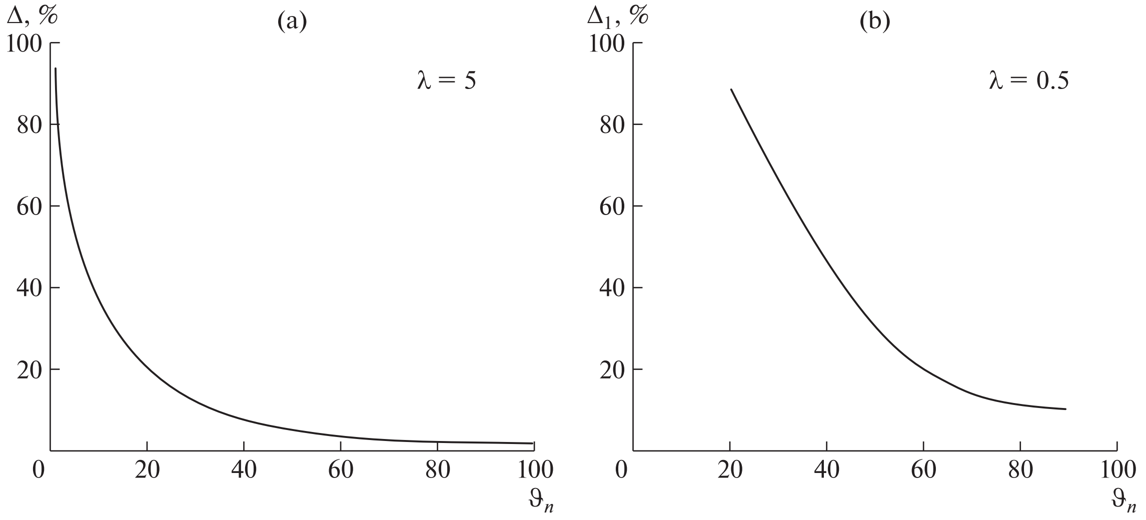

Fig. 2. Relative differences of risks Δ over value ϑ_n for (a) Δ and (b) Δ_1.

Risk (19) is uniformly larger than risk (17) of the optimal estimate (13) for $\theta > 0$. Consequently, the best unbiased estimate (18) is inadmissible in the class of all possible estimates. Any estimate different from estimate (13) is inadmissible in the class Ψ. However, as ϑ_n increases, risk (19) tends to risk (18).

In Fig. 2a we present the dependence of the relative difference of risks

$$\Delta(\vartheta_n, \lambda) = \frac{r_\theta(\theta_n^*) - r_\theta(\hat{\theta}_n^0)}{r_\theta(\theta_n^*)} \tag{20}$$

on ϑ_n at $\lambda = 5$.

We can see from the dependences provided above that at $\vartheta_n \leq 20$ the efficiency of estimate (13) is relatively high in comparison to estimate (18). As $\vartheta_n > 20$ increases, convergence of the risk of estimate (20) to the risk of optimal estimate (13) becomes extremely slow.

4. LOSS FUNCTION EQUAL TO ERROR MODULE

Suppose that the loss function is

$$f_n(\hat{\theta}_n, \theta) = |\hat{\theta}_n - \theta|.$$

By using expressions (9) and (10), we may show that the median of a posteriori PDF (7), where $\beta = 1$, is the optimal estimate, that is,

$$\Pi_n(\hat{\theta}_n^0) = \int\limits_0^{\hat{\theta}_n^0} W_n^{(1)}(\theta)\, d\theta = 0{,}5. \tag{21}$$

We use (7) ($\beta = 1$) and replace of variables $\theta^\lambda = q$, thus obtaining

$$\Pi_n(z) = \frac{1}{\Gamma(\vartheta_n - 1/\lambda)}\, z^\lambda \int\limits_0^{y_n} q^{\vartheta_n - 1/\lambda - 1} \mathbf{e}^{-q}\, dq = G_{2(\vartheta_n - 1/\lambda)}(2z^\lambda y_n), \tag{22}$$

where $G_a = \Gamma^{-1}(a/2) \int\limits_0^q \beta^{a/2-1} \mathbf{e}^{-\beta/2}\, d\beta / 2^{a/2}$ is the gamma distribution function with a degrees of freedom.

From expressions (21) and (22) we find

$$\hat{\theta}_n^0(y_n) = \left\{ \frac{g_{0,5}[2(\vartheta_n - 1/\lambda)]}{2y_n} \right\}^{1/\lambda}, \tag{23}$$

where $g_\varepsilon(a)$ is the ε-quantile of the gamma distribution with a degrees of freedom.

We may show that a posteriori risk of estimate (23) is determined by the expression

$$R_n^{(1)}(\hat{\theta}_n^0, y_n) = \frac{\Gamma(\vartheta_n) y_n^{-1/\lambda}[1 - G_{2\vartheta_n}\{g_{0,5}[2(\vartheta_n - 1/\lambda)]\}]}{\Gamma(\vartheta_n - 1/\lambda)}. \tag{24}$$

Applying (15), we suppose that

$$M_\theta Y_n^{-\alpha/\lambda} = \frac{\theta^\alpha \Gamma^2(\vartheta_n - \alpha/\lambda)}{\Gamma(\vartheta_n)}. \tag{25}$$

With account for expressions (23)–(25) we write

$$M_\theta \hat{\theta}_n^0(Y_n) = \frac{\theta g_{0,5}^{1/\lambda}[2(\vartheta_n - 1/\lambda)]\Gamma(\vartheta_n - 1/\lambda)}{\Gamma(\vartheta_n)},$$

$$r_\theta(\hat{\theta}_n^0) = \theta[1 - G_{2\vartheta_n}\{g_{0,5}[2(\vartheta_n - 1/\lambda)]\}]. \tag{26}$$

Thus, we can see from the above provided calculations that optimal estimate (23), as well as estimate (13), is biased. In this case the a posteriori average, as it follows from expression (12) at $\beta = 1$ and $m = 1$, coincides with the best unbiased estimate (18). Consequently, it appears to be uniformly worse than estimate (23). However, as ϑ_n increases, the differences between the risks of these estimates vanish.

We assess the efficiency of optimal estimate (23) in comparison with the best unbiased estimate (18) for finite values of ϑ_n.

The risk of estimate (18) is equal to

$$r_\theta(\theta_n^*) = 2\theta[G_{2(\vartheta_n - 1/\lambda)}(\rho_n) - G_{2\vartheta_n}(\rho_n)], \tag{27}$$

where $\rho_n = 2[\Gamma(\vartheta_n)/\Gamma(\vartheta_n - 1/\lambda)]^\lambda$.

As previously, we determine the efficiency with relation (20) and denote it by Δ_1.

We use expressions (26) and (27) and find the values Δ_1 in dependence on λ and ϑ_n. The dependence $\Delta_1 = f(\vartheta_n)$ at $\lambda = 0{,}5$ is given in Fig. 2b. We see from the presented dependence that, in contrast to the quadratic loss function, in this case the convergence of the risk of the best unbiased estimate (18) to the risk of the optimal estimate is very fast. For instance, if $\gamma_i = 1$, $i = \overline{1, n}$, then for achieving the relative difference in the risks below 10% we need just six observations ($n \geq 3$), whereas for quadratic loss function we need 50 observations.

5. INVARIANT LOSS FUNCTION

If the loss function is invariant to scale transformations (see expression (2)), then, according to the above discussed material, we need to choose PDF (7) as a posteriori PDF and set $\beta = 0$.

The optimal estimate in the class of estimate (3) is equivariant, that is,

$$r_\theta(\hat{\theta}_n^0) = \text{const}, \quad \theta > 0.$$

When the equality is true

$$g_n(|\hat{\theta}_n - \theta|/\theta) = \theta^{-\alpha}|\hat{\theta}_n - \theta|^\alpha, \tag{28}$$

the optimal estimates are the same as at power loss function $|\hat{\theta}_n - \theta|^\alpha$ [10]. For instance, at $\alpha = 1$ in expression (28) the optimal estimate is determiend by expression (23), and at $\alpha = 2$ it is determined by expression (13).

We may show that a posteriori risks of estimates (23) and (11) are independent of observations here. They coincide with the conditional risks and will be determined by the following relations:

$$R_n^{(1)}(\hat{\theta}_n^0) = r_\theta^{(1)}(\hat{\theta}_n^0) = [1 - G_{2\vartheta_n}\{g_{0,5}[2(\vartheta_n - 1/\lambda)]\}];$$

$$R_n^{(2)}(\hat{\theta}_n^0) = r_\theta^{(2)}(\hat{\theta}_n^0) = \left[1 - \frac{\Gamma^2(\vartheta_n - 1/\lambda)}{\Gamma(\vartheta_n)\Gamma(\vartheta_n - 2/\lambda)}\right]. \quad (29)$$

6. CONFIDENCE ESTIMATION

Because Y_n is a complete sufficient statistic [9], it follows from [10] that the lower $\theta_{\text{l}.n}^0$ and upper $\theta_{\text{u}.n}^0$ ε-confidence boundaries may be determined from the equalities

$$\int\limits_{\theta_{\text{l}.n}^0}^{\infty} W_n^{(0)}(\theta)\,d\theta = \varepsilon, \qquad \int\limits_{0}^{\theta_{\text{u}.n}^0} W_n^{(0)}(\theta)\,d\theta = \varepsilon.$$

Using expressions (7) and (29) and setting $\beta = 0$, we obtain

$$G_{2\vartheta_n}(2(\theta_{\text{l}.n}^0)^\lambda y_n) = 1 - \varepsilon, \quad G_{2\vartheta_n}(2(\theta_{\text{u}.n}^0)^\lambda y_n) = 1 - \varepsilon,$$

which leads to

$$\theta_{\text{l}.n}^0 = \left[\frac{g_{1-\varepsilon}(2\vartheta_n)}{2y_n}\right]^{1/\lambda}; \quad \theta_{\text{u}.n}^0 = \left[\frac{g_{\varepsilon}(2\vartheta_n)}{2y_n}\right]^{1/\lambda}. \quad (30)$$

We take into account that the joint distribution of independent random values $U_{s.1}, \ldots, U_{s.n}$ with the PDFs described by expression (3) has a monotonous likelihood ratio relative to the statistics Y_n. We use the results of [12] and can show that estimates (30) are uniformly most accurate confidence boundaries of the level ε. In its turn, it means that they are optimal in the sense of [12],

$$r_\theta(\theta_{\text{l}.n}^0) = M_\theta g_n^{(1)}(\theta_{\text{l}.n}^0, \theta) = \inf_{\theta_{\text{l}.n}} M_\theta g_n^{(1)}(\theta_{\text{l}.n}, \theta), \quad \theta > 0;$$

$$r_\theta(\theta_{\text{u}.n}^0) = M_\theta g_n^{(2)}(\theta_{\text{u}.n}^0, \theta) = \inf_{\theta_{\text{u}.n}} M_\theta g_n^{(2)}(\theta_{\text{u}.n}, \theta), \quad \theta > 0,$$

where the function $g_n^{(1)}(\theta_{\text{l}.n}^0, \theta) = 0$ for $\theta_{\text{l}.n} \geq \theta$ and does not increase in $\theta_{\text{l}.n}$ in the domain $\theta_{\text{l}.n} < 0$, $g_n^{(2)}(\theta_{\text{u}.n}, \theta) = 0$ for $\theta_{\text{u}.n} \leq 0$ and does not decrease in $\theta_{\text{u}.n}$ in the domain $\theta_{\text{u}.n} > 0$, and the infimums $\inf_{(\cdot)}$ are chosen by the classes of all possible estimates satisfying the conditions

$$P_\theta(\theta_{\text{l}.n}(U^n) < \theta) \geq \varepsilon; \quad P_\theta(\theta_{\text{u}.n}(U^n) < \theta) \geq \varepsilon.$$

Thus, the ε-confidence boundaries found with the Bayesian formalism appear to be optimal at particularly general loss functions not always satisfying the properties described in (2) and (5) in the classes of all possible estimates of levels not less than ε.

As functions $g_n^{(1)}(\cdot)$, $g_n^{(2)}(\cdot)$ it is purposeful to choose

$$g_n^{(1)}(\theta_{\text{l}.n}, \theta) = \begin{cases} g_n(\theta - \theta_{\text{l}.n}), & \theta_{\text{l}.n} < \theta; \\ 0, & \theta_{\text{l}.n} \geq \theta, \end{cases} \qquad g_n^{(2)}(\theta_{\text{u}.n}, \theta) = \begin{cases} g_n(\theta_{\text{u}.n} - \theta), & \theta_{\text{u}.n} > \theta; \\ 0, & \theta_{\text{u}.n} \leq \theta, \end{cases}$$

where $g_n(y)$ is some decreasing function of y.

Here, if $g_n(y) = y$, then the risks $r_\theta(\theta_{\text{l}.n}^0)$ and $r_\theta(\theta_{\text{u}.n}^0)$ are equal to the errors of underestimating and overestimating, which are determined by

$$r_\theta(\theta_{\text{l}.n}^0) = \theta\left\{\varepsilon - \Gamma\left(\vartheta_n - \frac{1}{\lambda}\right) g_{1-\varepsilon}^{1/\lambda}(2\vartheta_n)\frac{[1 - G_{2(\vartheta_n - 1/\lambda)}\{2g_{1-\varepsilon}2(\vartheta_n)\}]}{\Gamma(\vartheta_n)}\right\},$$

$$r_\theta(\theta_{\text{u}.n}^0) = \theta\left\{\Gamma\left(\vartheta_n - \frac{1}{\lambda}\right) g_{\varepsilon}^{1/\lambda}(2\vartheta_n)\frac{[G_{2(\vartheta_n - 1/\lambda)}\{2g_{\varepsilon}2(\vartheta_n)\}}{\Gamma(\vartheta_n) - \varepsilon}\right\}.$$

In the special case when $\gamma_i = 1$, $i = \overline{1, n}$, the above obtained estimates correspond to the estimation of the parameter of the Weibull PDF. If $\lambda = 1$ here, then the obtained estimates are optimal estimates of intensity of the Poisson stream.

7. CONCLUSIONS

We showed that, in order to describe the probabilistic characteristics of motion intensity of extended objects, we may use the generalized Weibull distribution law. We performed the optimal estimation of the scale parameter of the generalized Weibull probability density function using the criterion of minimum of the conditional risk. The optimal estimates of intensity of an inhomogeneous process (of the envelope of the signal reflected from the extended object) with the generalized Weibull distribution of the inter-event times (the intervals between the appearances of the envelope signals reflected from extended objects at the input of the speed meter) were obtained including determination of the equivariant estimates for invariant loss functions. We conducted comparative analysis with the best unbiased estimate for different loss functions: quadratic function, function equal to the error module, and invariant one, and for the confidence estimation. We determined the conditional risk of estimating the scale parameter and the conditions of its minimization, including the case of using the function of a posteriori risk. We showed that the optimal estimate of the parameter for the quadratic loss function and for the loss function equal to the error module is biased. In addition to that, for the loss function equal to the error module, the convergence of the risk of the best unbiased estimate to the risk of optimal estimate is faster compared to the quadratic loss function, which leads to considerable reduction in the number of observations. The optimal estimates coincide for the invariant and power loss functions. We obtained the lower and upper ε-confidence boundaries which appear to be optimal for particularly general loss functions and determined the conditions under which the obtained estimates correspond to the estimation of the Weibull distribution parameter. It was noted that the considered problem has a wide spectrum of practical applications.

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