Determination of the Steepness of the Discriminatory Characteristic for the Meter of non-Energy Parameters of the Signal

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***Abstract*—The simultaneous influence of multiplicative and additive noises on the steepness of the discriminative characteristic of a two-channel discriminator is analyzed. A coherent signal and a sequence of incoherent pulses are used as input signals. It is assumed that, at a relatively low level of noise, the tracking meter is considered as a linear system. Conditions are obtained when systematic measurement errors are absent. The expressions for determining the steepness of the discriminative characteristic affected by multiplicative noise depending on the additive noise level are obtained. It is shown that multiplicative noise has a greater influence on the steepness of the discriminative characteristic when the level of additive noise is higher. Coefficients determining the normalizing effect of the automatic gain control are obtained. It is shown that the type of the input signal does not change the physical nature of the influence of multiplicative and additive noises on the steepness of the discriminative characteristic.**

Keywords—tracking meter, additive noise, multiplicative noise, measurement error, steepness of discriminative characteristic, coherent signal, incoherent pulse, automatic gain control

I. Introduction

In information-measuring systems for various purposes, tracking meters have been widely used. At the same time, they are often designed to measure non-energy parameters of a useful signal, for example, frequency, initial phase, etc. In publications [1-7, etc.] the study of some features of their work is given. The authors in publication [8] showed how the determination of the spectral density (SD) of the mismatch signal of the discriminator, as well as the steepness of the discriminatory characteristic (DC) of the latter, makes it possible to obtain an estimate of the measurement accuracy of one information parameter of the signal. At the same time, both multiplicative (MN) and additive (AN) noises acted on the signal at the same time [9-15, etc.]. In addition, the above-mentioned publication provides a generalized structure diagram of a tracking meter, as well as a functional diagram of a two-channel discriminator, the latter being intended to be used in the present work.

The purpose of the present work is to determine the steepness of the DC of a two-channel discriminator. At the same time, finding the values of the of the automatic gain control (AGC) normalizing effect coefficients under the influence of coherent and incoherent input signal is of significant interest for the study [16].

Further, we will rely on the approaches outlined by the authors in [8], describing the performance of a useful signal parameter meter under the influence of AN and MN.

II. Peculiarities of Operation of the Two-Channel Discriminator of the Tracking Meter

Let us use the presented functional diagram of a discriminator with two channels of signal processing, mismatched by the measured parameter l (Fig. 1). Let the input signal $u_{in}(t)$ be under the simultaneous influence of AN and MN or only by the former of them. Each of the discriminator channels is formed by a multiplier, a filter (F) and a quadratic envelope detector (QED). The filter has a frequency response $\dot{g}_F(\omega)$. The multipliers receive an input signal and a reference signal $u_0(l = \ldots)$. Note that the reference signal will have a value proportional to the previously determined value of the parameter l.

As can be seen from the functional diagram, the reference signals are shifted by $\pm\Delta_l$ relative to the value of the parameter l^* measured at a given time. Also, the reference signal depends on the measurement error $\varepsilon = l^* - l_0$, which in this case is a fluctuation error. The latter statement is substantiated in [8], where it is shown that the fluctuation measurement error occurs under the influence of AN and MN on the useful signal.

The difference of the squares of the signal envelopes at the output of the filters forms the discriminator mismatch signal (error signal) $\chi(\varepsilon, t)$.

If the level of noise acting on the discriminator is relatively low, then the discriminatory characteristic by measurement error is ε considered linear [8]. At the same time, the tracking meter itself will be considered a linear system by the measured parameter. Then the discriminator signal $\chi(\varepsilon, t)$ can be represented like this [17]:

$$\chi(\varepsilon,t) \simeq \varepsilon \frac{d}{d\varepsilon}\overline{\chi(\varepsilon,t)}\Big|_{\varepsilon=0} + G_\chi^{0.5} n_0(t) = \varepsilon k_d + G_\chi^{0.5} n_0(t), \quad (1)$$

where G_χ is the SD of fluctuations of the discriminator output signal $\chi(\varepsilon, t)$ at zero frequency when $\varepsilon = 0$, $k_d = \varepsilon \frac{d}{d\varepsilon}\overline{\chi(\varepsilon,t)}\Big|_{\varepsilon=0}$ is the steepness of the DC, $n_0(t)$ is white noise with a unit SD.

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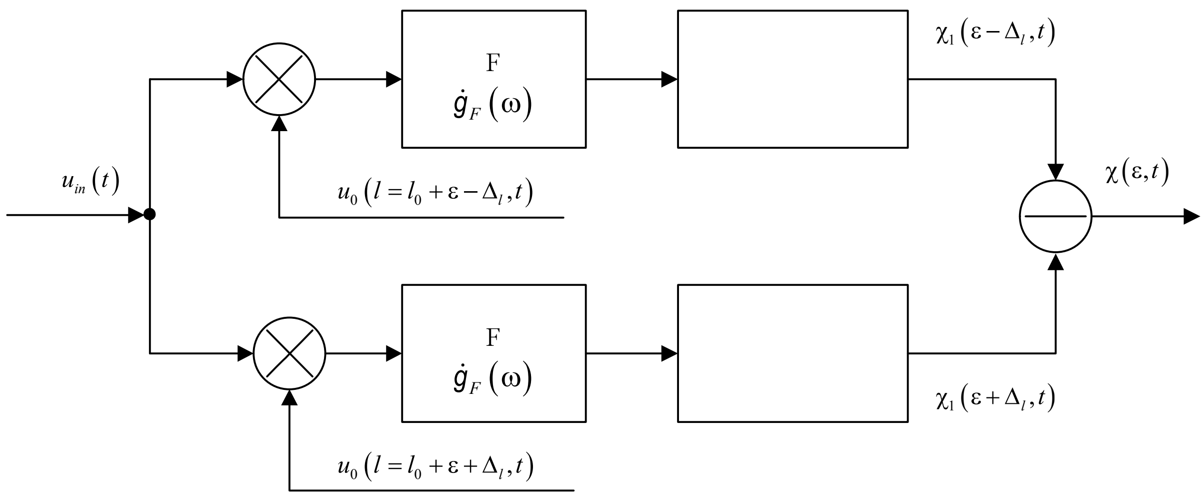

Fig. 1. Functional diagram of a two-channel discriminator [8]

Let us also briefly focus on the features of AGC in tracking meters of signal parameters. The purpose of using AGC in the meter is to maintain a constant value of the average power level or envelope of the input signal $u_{in}(t)$ of the discriminator. The introduction of the normalizing coefficient k_{AGC} will allow to take into account the influence of the AGC scheme only when the power of the input signal changes slowly. The time of change of said power, in turn, is determined by the time constant of the linear circuits of the discriminator. The k_{AGC} coefficient is often proportional to the input signal power $u_{in}(t)$. In a two-channel discriminator circuit, the k_{AGC} coefficient will be proportional to the sum of the squares of the signal envelopes at the output of the filters.

Apparently, the AGC will have a modulating effect on the steepness of the DC.

Since a quadratic envelope detector is selected in the diagram in Fig. 1, the following formula can be written to calculate the normalizing effect of the automatic gain control on the steepness of the DC [8]:

$$k_{d.AGC} = \frac{k_d}{k_{AGC}\left[u_{in}(t)\right]}.$$

We will also use the coefficients, introduced earlier in [8], which characterize the influence of the AGC:

– $k_{d.n.AGC}$ is the nominal steepness of the DC in the absence of AN and MN (the index "n" denotes the nominal values of other parameters below as well), taking into account the normalizing influence of automatic gain control;

– $k_{d.n.}$ is the nominal steepness of the DC in the absence of AN and MN without taking into account the normalizing influence of automatic gain control;

– $k_{d.0}$ is the steepness of the DC in the presence of AN only and without taking into account the normalizing influence of automatic gain control;

– $k_{d.0.AGC}$ is the steepness of the DC in the presence of AN only, taking into account the normalizing influence of automatic gain control;

– $k_{n.AGC}$ is a nominal normalizing coefficient, taking into account the influence of automatic gain control.

The following apparent expressions linking the given coefficients can be given [8]:

$$k_{d.0.AGC} = k_{d.0}/k_{0.AGC}; \quad k_{d.n.AGC} = k_{d.n}/k_{n.AGC}.$$

We assume that the signal at the input of the linear circuits of the discriminator has a unit power under the condition of the normalizing influence of the automatic gain control.

As will be shown below, the effect of MN on the measurement accuracy of the parameter l can be estimated by calculating the steepness of the DC (k_d, $k_{d.0}$) and the above coefficients (k_{AGC}. $k_{d.0.AGC}$, $k_{d.n.AGC}$), as well as the SD of fluctuations at the discriminator output when $\varepsilon = 0$. However, the definition of the latter is not the subject of this publication.

Let us consider two practically important cases of determining the steepness of the DC: with a coherent input signal and with an input signal in the form of a sequence of incoherent pulses.

III. Steepness of the Discriminative Characteristic with a Coherent Input Signal

In accordance with the functional diagram shown in Fig. 1, the output signal of the discriminator is written as

$$\chi(\varepsilon,\tau) = \chi_1(\varepsilon - \Delta_l, \tau) - \chi_2(\varepsilon + \Delta_l, \tau), \tag{1}$$

where the functions $\chi_{1.2}(\varepsilon \pm \Delta_l, \tau)$ in stationary mode are equal to

$$\chi_{1.2}(\varepsilon \pm \Delta_l, \tau) = \frac{1}{8}\mathrm{Re}\Big\{\iint_{-\infty}^{\infty}\left[\dot{M}(t_1)\dot{U}(t_1,l_0) + \dot{N}(t_1)\right]\times$$
$$\times\left[\dot{M}(t_2)\dot{U}(t_2,l_0) + N^*(t_2)\right]\dot{H}(\tau - t_1)H^*(\tau - t_2)\times \tag{2}$$
$$\times U_0^*(t_1, l_0 - \varepsilon \mp \Delta_l)\dot{U}_0(t_2, l_0 - \varepsilon \mp \Delta_l)\,dt_1 dt_2\Big\},$$

where $\dot{M}(t)$ is noise modulation function (NMF), $\dot{H}(\tau - t)$ is a complex envelope of the pulse response of the filter with a frequency response $\dot{g}_F(\omega)$, $\dot{U}_0(t, l_0 + \varepsilon \mp \Delta_l)$ is a complex envelope of the reference signal of a unit energy, $\dot{U}(t, l_0) = \sqrt{E}\dot{U}_0(t, l_0)$ is a complex envelope of the normalized received signal, E is the energy of the received signal, $\dot{N}(t)$ is a complex envelope of AN, U_0 is amplitude of the reference signal.

The sign * above indicates complex-conjugate values.

In expression (2), a rapidly oscillating term is discarded, which is filtered out in the inertial circuits of quadratic detectors included in both channels of the discriminator.

We find the steepness of the DC and the coefficients that determine the normalizing effect of the AGC, assuming that the AN is white with a SD N_0. For a coherent input signal, the average value of the discriminator output signal in the stationary mode is equal to

$$\begin{aligned}\overline{\chi(\varepsilon,\tau)} &= \frac{E}{2}\alpha_0^2\left[\left|\dot{\lambda}(\varepsilon-\Delta_l)\right|^2-\left|\dot{\lambda}(\varepsilon+\Delta_l)\right|^2\right]+\\ &+\frac{E}{4\pi}\int_{-\infty}^{\infty}G_V(\Omega)\left[\left|\dot{\lambda}(\varepsilon-\Delta_l,\Omega)\right|^2-\left|\dot{\lambda}(\varepsilon+\Delta_l,\Omega)\right|^2\right]d\Omega,\end{aligned} \tag{3}$$

where

$$\begin{aligned}&\dot{\lambda}(\varepsilon-\Delta_l,\Omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\dot{g}_F(\omega)\rho^*(\varepsilon-\Delta_l,\Omega-\omega)d\omega;\\ &\dot{\lambda}(\varepsilon-\Delta_l)=\dot{\lambda}(\varepsilon-\Delta_l,\Omega-\omega);\\ &\dot{\rho}(\varepsilon-\Delta_l,\Omega-\omega)=\\ &=\frac{1}{2}\int_{-\infty}^{\infty}\dot{U}_0(t)U_0^*(t,\varepsilon-\Delta_l)\exp\{j(\Omega-\omega)t\}dt.\end{aligned}$$

The coefficient K_{AGC} taking into account (3) is equal to

$$\begin{aligned}k_{AGC} &= \overline{\chi_1(\varepsilon,\tau)}+\overline{\chi_2(\varepsilon,\tau)}=\\ &=\frac{E}{2}\alpha_0^2\left[\left|\dot{\lambda}(\varepsilon-\Delta_l)\right|^2+\left|\dot{\lambda}(\varepsilon+\Delta_l)\right|^2\right]+\\ &+\frac{E}{4\pi}\int_{-\infty}^{\infty}G_V(\Omega)\left[\left|\dot{\lambda}(\varepsilon-\Delta_l,\Omega)\right|^2+\left|\dot{\lambda}(\varepsilon+\Delta_l,\Omega)\right|^2\right]d\Omega+\\ &+N_0/2\pi\int_{-\infty}^{\infty}g_F^*(\omega)\dot{\lambda}(0,\omega)d\omega.\end{aligned} \tag{4}$$

It is taken into account that $\overline{\dot{N}(t_1)N^*(t_2)}=2N_0\delta(t_1-t_2)$, $\overline{\dot{N}(t_1)\dot{N}(t_2)}=0$.

To determine the conditions for the absence of systematic measurement errors, we will analyze the expression for the average value of the discriminator output signal (3). In general, this requirement is met if $\overline{\chi(0,\tau)}=0$.

Record the signal at the discriminator output in the absence of multiplicative noise

$$\overline{\chi(\varepsilon,\tau)}=\frac{E}{2}\left|\dot{\lambda}(\varepsilon-\Delta_l)\right|^2-\left|\dot{\lambda}(\varepsilon+\Delta_l)\right|^2. \tag{5}$$

It can be seen from (5) that in the presence of AN only and with the same frequency characteristics of filters $\dot{g}_F(\omega)$ in both channels of the discriminator, the estimation bias, a systematic measurement error, will be absent ($\overline{\chi(0,\tau)}=0$) under the condition

$$\left|\dot{\lambda}(-\Delta_l)\right|^2-\left|\dot{\lambda}(\Delta_l)\right|^2. \tag{6}$$

Condition (6) is satisfied, for example, if $\dot{g}_F(\omega)$ is a function that is even with respect to the average frequency of the filter. Further, we will assume that such filters are used in the discriminator.

In the presence of AN and MN, there is no systematic measurement error, if an additional condition is met

$$\int_{-\infty}^{\infty}G_V(\Omega)\left[\left|\dot{\lambda}(-\Delta_l,\Omega)\right|^2-\left|\dot{\lambda}(\Delta_l,\Omega)\right|^2\right]d\Omega=0. \tag{7}$$

Since the condition $\left|\dot{\lambda}(-\Delta_l,\Omega)\right|^2=\left|\dot{\lambda}(\Delta_l,-\Omega)\right|^2$ is fulfilled for most real signals and meters, in the presence of MN, there is no systematic measurement error only when $G_V(\Omega)=G_V(-\Omega)$, that is, the spectrum of fluctuations of the NMF is symmetric with respect to Ω = 0. The latter condition may not be fulfilled in some cases, for example, if there is a connection between amplitude and phase distortions.

Thus, MN can lead to the systematic measurement error even in the case when there are no such errors in the presence of only AN.

In the absence of systematic measurement errors, when conditions (6) and (7) are met, the steepness of the DC is equal to

$$\begin{aligned}k_d &= \alpha_0^2E\frac{d}{d\varepsilon}\left|\dot{\lambda}(\varepsilon-\Delta_l)\right|^2_{\varepsilon=0}+\\ &+\frac{E}{2\pi}\int_{-\infty}^{\infty}G_V(\Omega)\frac{d}{d\varepsilon}\left|\dot{\lambda}(\varepsilon-\Delta_l,\Omega)\right|^2_{\varepsilon=0}d\Omega.\end{aligned} \tag{8}$$

It is shown in [8] that $k_{d.n}=k_{d.0}$, and an expression can be written to determine the variance of measurement errors of the parameter l under the simultaneous influence of AN and MN for various filter circuits of the discriminator:

$$\begin{aligned}\sigma_l^2 &= \sigma_{l.0}^2\frac{G_\chi}{G_{\chi.0}}\frac{k_{d.0}^2}{k_d^2}\frac{k_{0.AGC}}{k_{AGC}}=\sigma_{l.0}^2\eta_{l.M.1};\\ \sigma_l^2 &= \sigma_{l.0}^2\frac{G_\chi}{G_{\chi.0}}\frac{k_{d.0}^2}{k_d^2}\frac{1+(k_{n.AGC}/k_{AGC})(k_d/k_{d.0})}{1+k_{n.AGC}/k_{0AGC}}=\sigma_{l.0}^2\eta_{l.M.2};\\ \sigma_l^2 &= \sigma_{l.0}^2\frac{G_\chi}{G_{\chi.0}}\frac{k_{d.0}^2}{k_d^2}\frac{1+(k_{n.AGC}/k_{AGC})(k_d/2k_{d.0})}{1+k_{n.AGC}/2k_{0AGC}}=\sigma_{l.0}^2\eta_{l.M.3},\end{aligned} \tag{9}$$

where $\sigma_{l.0}^2$ is the variance of the measurement errors of the parameter l in the presence of only AN, G_χ and $G_{\chi 0}$ is the SD of the discriminator output signal, respectively, with simultaneous exposure to AN and MN and exposure to only AN, k_d and $k_{d.0}$ is the steepness of the DC also, respectively, with simultaneous exposure of AN and MN and exposure to only AN, $\eta_{l.M1}$ is a coefficient that takes into account the deterioration of the measurement accuracy of the parameter l under the influence of MN.

Note that in formulas (9) above, the SD of the output signal and the steepness of the DC do not take into account the normalizing effect of automatic gain control.

The ratio $k_d/k_{d.0}$ included in (9), taking into account (8), will be equal to

$$\begin{aligned}\frac{k_d}{k_{d.0}} &= \alpha_0^2+\left[2\pi\frac{d}{d\varepsilon}\left|\dot{\lambda}(\varepsilon-\Delta_l)\right|^2_{\varepsilon=0}\right]^{-1}\times\\ &\times\int_{-\infty}^{\infty}G_V(\Omega)\frac{d}{d\varepsilon}\left|\dot{\lambda}(\varepsilon-\Delta_l,\Omega)\right|^2_{\varepsilon=0}d\Omega.\end{aligned} \tag{10}$$

Provided that there is no systematic measurement error for the relations $k_{AGC}/k_{0.AGC}$, $k_{AGC}/k_{n.AGC}$, $k_{0.AGC}/k_{n.AGC}$ included in (9), taking into account (4), we get (when ε = 0)

$$\frac{k_{AGC}}{k_{0.AGC}}=\left\{\left|\dot{\lambda}(\Delta_l)\right|^2+\frac{N_0}{2\pi E}\int_{-\infty}^{\infty}g_F^*(\Omega)\dot{\lambda}(0,\Omega)d\Omega\right\}^{-1}\times$$
$$\times\left[\alpha_0^2\left|\dot{\lambda}(\Delta_l)\right|^2+\frac{1}{2\pi}\int_{-\infty}^{\infty}G_V(\Omega)\left|\dot{\lambda}(\Delta_l,\Omega)\right|^2d\Omega+\right.$$
$$\left.+\frac{N_0}{2\pi E}\int_{-\infty}^{\infty}g_F^*(\Omega)\dot{\lambda}(0,\Omega)d\Omega;\right.$$
$$\frac{k_{AGC}}{k_{n.AGC}}=\left|\dot{\lambda}(\Delta_l)\right|^{-2}\left[\alpha_0^2\left|\dot{\lambda}(\Delta_l)\right|^2+\right.$$
$$+\frac{1}{2\pi}\int_{-\infty}^{\infty}G_V(\Omega)\left|\dot{\lambda}(\Delta_l,\Omega)\right|^2d\Omega+$$
$$\left.+\frac{N_0}{2\pi E}\int_{-\infty}^{\infty}g_F^*(\Omega)\dot{\lambda}(0,\Omega)d\Omega\right];$$
$$\frac{k_{0.AGC}}{k_{n.AGC}}=\left|\dot{\lambda}(\Delta_l)\right|^{-2}\times$$
$$\times\left[\left|\dot{\lambda}(\Delta_l)\right|^2+\frac{N_0}{2\pi E}\int_{-\infty}^{\infty}g_F^*(\Omega)\dot{\lambda}(0,\Omega)d\Omega\right].$$

From expression (10), it is apparent that the presence of MN leads to a decrease in the steepness of the DC and within the limit when $\Delta\Omega_M\rightarrow\left[G_V(0)\rightarrow 0\right]$ at very broadband MN, the ratio $(k_d/k_{d.0})$ tends to α_0^2, that is, it decreases by a factor of $1/\alpha_0^2$. Taking into account the normalizing action of the automatic gain control, as the spectrum width of the NMF increases within the limit when $\Delta\Omega_M\rightarrow\infty$ the ratio $(k_d/k_{d.0})(k_{0.AGC}/k_{AGC})$ reaches the value

$$\alpha_0^2\left[\left|\dot{\lambda}(\Delta_l)\right|^2+\varsigma\frac{N_0}{E}\right]\left[\alpha_0^2\left|\dot{\lambda}(\Delta_l)\right|^2+\varsigma\frac{N_0}{E}\right]^{-1},$$

where $\varsigma=\frac{1}{2\pi}\int_{-\infty}^{\infty}g_F^*(\Omega)\dot{\lambda}(0,\Omega)d\Omega$ is a constant coefficient determined by the characteristics of the linear circuits of the discriminator.

The latter expression shows the dependence of the change in the steepness of the DC under the influence of MN on the level of AN. It shows that the influence of MN on the steepness of the discriminative characteristic is greater at a high level of AN. However, as will be seen further, the influence of MN on the SD of the discriminator output signal and on the variance of measurement errors is greater the lower the level of AN. This is due to the fact that a decrease in the steepness of the DC, as follows from (5) [8], for example, leads to a decrease in the effective width of the frequency response of the meter, that is, to a decrease in the variance of fluctuation measurement errors.

With a low level of AN, the above-mentioned decrease will be smaller; therefore, an increase in the SD of the discriminator output signal due to the action of MN will have a greater effect on measurement errors than a decrease in the steepness of the DC.

IV. The Steepness of the Discriminative Characteristic with the Input Signal in the Form of a Sequence of Incoherent Pulses

Suppose the received signal represent a sequence of incoherent pulses, a sequence of pulses with random initial phases $\varphi_{0.k}$, while the ratio of the pulse repetition period T_r to the duration of one pulse T (pulse ratio) is much greater than unity, which is always fulfilled in the practice of using incoherent pulse sequences.

If $U_1(t)\exp\{j\varphi_0\}$ is a complex envelope of one pulse of the sequence, then the complex envelope of the entire received sequence is described

$$\dot{U}(t)=\sum\nolimits_k\dot{U}_1(t-kT_r)\exp\{j\varphi_{0.k}\}. \tag{11}$$

To find the average value of the discriminator output signal, it is necessary to substitute the received signal in the form (11) in (1) and (2). Further presenting the mixed moment $\overline{\dot{M}(t_1)M^*(t_2)}$ and a pulse transition function of the filters in the form of their Fourier transform and taking into account the relation [18]

$$\sum\nolimits_{k=-\infty}^{\infty}\exp\{jkT_r(\omega_1-\omega_2)\}=$$
$$=\frac{1}{T_r}\sum\nolimits_{k=-\infty}^{\infty}\delta\left(\frac{2\pi k}{T_r}-\omega_1+\omega_2\right)$$

after rather cumbersome transformations, write such an expression that allows you to determine the average value of the signal at the output of the discriminator:

$$\overline{\chi(\varepsilon,\tau)}=\frac{1}{T_r}\sum\nolimits_{m=-\infty}^{\infty}\exp\left\{j\tau\frac{2\pi m}{T_r}\right\}\times$$
$$\times\frac{E}{8\pi^2}\iint_{-\infty}^{\infty}G_M(\Omega)\dot{g}_F\left(\omega+\frac{2\pi m}{T_r}\right)g_F^*(\omega)\times$$
$$\times\left[\rho_1^*\left(\varepsilon-\Delta_l,\Omega+\omega+\frac{2\pi m}{T_r}\right)\dot{\rho}_1(\varepsilon-\Delta_l,\Omega+\omega)-\right. \tag{12}$$
$$\left.-\rho_1^*\left(\varepsilon+\Delta_l,\Omega+\omega+\frac{2\pi m}{T_r}\right)\dot{\rho}_1(\varepsilon+\Delta_l,\Omega+\omega)\right]d\Omega d\omega,$$

where

$\dot{\rho}_1(\varepsilon-\Delta_l,\Omega+\omega)=\frac{1}{2}\int_0^{Tr}\dot{U}_0(t,\varepsilon-\Delta_l)U_0^*(t)\exp\{j\Omega t\}dt$ is an autocorrelation function of a single signal period, which for the case under consideration, when the signal duration is much smaller than the repetition period T_r, coincides with the autocorrelation function of a single signal $\dot{\rho}_1(\varepsilon-\Delta_l,\Omega)$, $G_M(\Omega)$ is an energy spectrum of the NMF.

In accordance with (12), the average value of the discriminator output signal in stationary mode depends on time τ. This dependence is due to the fact that the received signal is an incoherent sequence of long-range pulses with a repetition period T_r. In this case, the duration of a single pulse of a sequence of pulses will determine the time constant of the linear circuits of the discriminator (namely: they must be equal).. Since the time constant of smoothing circuits at the output of the discriminator is much larger than T_r, it is advisable to consider the time-averaged characteristics of the signal at the output of the discriminator.

Taking into account

$$\frac{1}{T_r}\int_{T_r/2}^{T_r/2}\exp\left\{j\frac{2\pi m}{T_r}\tau\right\}d\tau=\frac{\sin\pi m}{\pi m}=\delta_m,$$

where $\delta_m = 1$ when $m = 0$, $\delta_m = 0$ when $m \neq 0$, and representing the energy spectrum of the NMF as $G_M(\Omega)=\alpha_0^2\delta(\Omega)+G_V(\Omega)$, from (12) the signal averaged over the ensemble and time at the discriminator output will be as follows:

$$\overline{\langle\chi(\varepsilon,\tau)\rangle}=\frac{\alpha_0^2E}{4\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\Big[\left|\dot\rho_1(\varepsilon-\Delta_l,\Omega)\right|^2-$$
$$-\left|\dot\rho_1(\varepsilon+\Delta_l,\Omega)\right|^2\Big]d\Omega+\frac{E}{4\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\times \qquad (13)$$
$$\times\Big[\left|\sigma_{s.1}^2(\varepsilon-\Delta_l,\Omega)\right|^2-\left|\sigma_{s.1}^2(\varepsilon+\Delta_l,\Omega)\right|^2\Big]d\Omega,$$

where $G_F(\Omega)=\left|\dot g_F(\Omega)\right|^2$,

After transformations similar to those carried out above, when calculating the function $\overline{\chi(\varepsilon,\tau)}$, in the absence of a systematic measurement error for coefficients that take into account the influence of AGC, we obtain (if $\varepsilon = 0$) (16).

The analysis of expressions (15) and (16) shows that the same conclusions regarding the influence of AN and MN on the steepness of the DC which are given in "Steepness of the discriminative characteristic with a coherent input signal" for coherent signals hold for signals in the form of an incoherent sequence of pulses.

Thus, the type of the received signal does not change the physical nature of the influence of AN and MN on the steepness of the DC.

V. Conclusions

The analysis of the effect of AN and MN on the steepness of the discriminatory characteristic of the discriminator with a coherent input signal, as well as with an input signal in the form of a sequence of incoherent pulses, is carried out.

$$\frac{k_{AGC}}{k_{0.AGC}}=\left[\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega+\frac{N_0}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)d\Omega\right]^{-1}\left[\frac{\alpha_0^2E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega+\right.$$
$$\left.+\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\sigma_{s.1}^2(\Delta_l,\Omega)d\Omega+\frac{N_0}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)d\Omega\right],$$
$$\frac{k_{AGS}}{k_{n.AGS}}=\left[\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega\right]^{-1}\left[\frac{\alpha_0^2E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega+\right. \qquad (16)$$
$$\left.+\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\sigma_{s.1}^2(\Delta_l,\Omega)d\Omega+\frac{N_0}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)d\Omega\right];$$
$$\frac{k_{0.AGC}}{k_{n.AGC}}=\left[\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega\right]^{-1}\left[\frac{E}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)\left|\dot\rho_1(\Delta_l,\Omega)\right|^2d\Omega+\frac{N_0}{2\pi T_r}\int_{-\infty}^{\infty}G_F(\Omega)d\Omega\right].$$

$$\sigma_{s.1}^2(\varepsilon-\Delta_l,\Omega)=2\sigma_s^2(\varepsilon-\Delta_l,\Omega)=$$
$$=\frac{2}{2\pi}\int_{-\infty}^{\infty}G_V(\Omega)\left|\dot\rho_1(\varepsilon-\Delta_l,\omega+\Omega)\right|^2d\Omega;$$

$\sigma_s^2(\varepsilon-\Delta_l,\omega)$ is a variance of the noise component of the signal distorted by MN at the output of the filter matched with the received signal.

The analysis of expression (13) shows that for the signal in the form of a systematic measurement error due to MN, we get

$$\left|\dot\rho_1(-\Delta_l,\Omega)\right|^2=\left|\dot\rho_1(\Delta_l,-\Omega)\right|^2;\quad G_V(\Omega)=G_V(-\Omega);$$
$$G_F(\Omega)=G_F(-\Omega), \qquad (14)$$

that is, they actually coincide with the corresponding conditions for a coherent signal.

In the absence of a systematic measurement error, if the conditions (14) are met, we have

$$\frac{k_d}{k_{d.0}}=\alpha_0^2+\frac{\int_{-\infty}^{\infty}G_F(\Omega)\frac{d}{d\varepsilon}\left[\sigma_{s.1}^2(\varepsilon-\Delta_l,\Omega)\right]_{\varepsilon=0}d\Omega}{\int_{-\infty}^{\infty}G_F(\Omega)\frac{d}{d\varepsilon}\left[\dot\rho(\varepsilon-\Delta_l,\Omega)\right]_{\varepsilon=0}^2d\Omega}. \qquad (15)$$

The case of the effect of white AN on the coherent input signal, for which the steepness of the DC is determined, is considered. Also found are values of coefficients under the regulating action of AGC. A condition is obtained under which a systematic measurement error will be absent both under the influence of AN only and AN and MN together. It is shown, at the same time, that MN can cause systematic measurement errors even for cases when these errors are absent under the influence of only AN. It is shown that MN has a greater effect on the steepness of the DC at a higher level of AN. It is also shown that a decrease in the steepness of the DC due to the influence of MN at a low level of AN has less effect on measurement errors than an increase in the SD of the signal at the output of the discriminator.

It has been found out that the average value of the signal at the output of the discriminator (mismatch signal) of the discriminator at the input sequence of incoherent pulses in the stationary mode depends on the time constant of linear circuits of the discriminator. The effect of MN on the input sequence of incoherent pulses in the form of measurement bias will be similar to the conditions of noise impact on the coherent signal. Expressions are defined for determination of normalizing values of coefficients in AGC action circuit. It has been shown that the conclusions regarding the effect of AN and MN on the slope of the DC are equally applicable for both input coherent signals and input sequences of incoherent pulses. In other words, the physical essence of the influence of AN and MN on the steepness of the DC does not depend on the type of received signal.

The analysis of the change in the slope of the DC under the combined influence of AN and MN at various input signals represents the scientific novelty of the work.

The obtained mathematical expressions that make it possible to determine the steepness of the discriminator DC, as well as the values of the AGC normalizing coefficients under the influence of AN and MN on the coherent and incoherent (in this work in the form of a sequence of incoherent pulses) input signals constitute the practical significance of the study.

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